Conformal Fixed Point of SU(3) Gauge Theory with 12 Fundamental Fermions in the Twisted Polyakov Loop Scheme

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and

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Plan of my talk

- Introduction
- Twisted Polyakov loop (TPL) scheme
- Nf=12 results
- Summary and outlook

Introduction

Motivation

- **Search of non-trivial IR fixed point (IRFP) in N_f flavor QCD**
  - For dynamical EW breaking in (walking) technicolor model
    - One of the candidates of BSM (revival from 80's…)
    - Predict dynamical symmetry breaking in strongly interacting gauge theory, analogy with QCD
    - To get rid of large FCNC, S-parameter and small quark mass

      1. Nearly conformal theory

      2. Large anomalous mass dimension

      Does the naïve extension of QCD to large flavor theory realize the above conditions ? → non-perturbative study is important !

Holdom (1986), Yamawaki (1986)
Appelquist (1986), …

  - For phase structure

    Chiral broken phase and confinement phase will be different or not ?

    → perception of phase structure of QCD

Gardi and Karliner (1998),
Miransky and Yamawaki (1997),
Ryttov and Sannino (2008)

Motivation

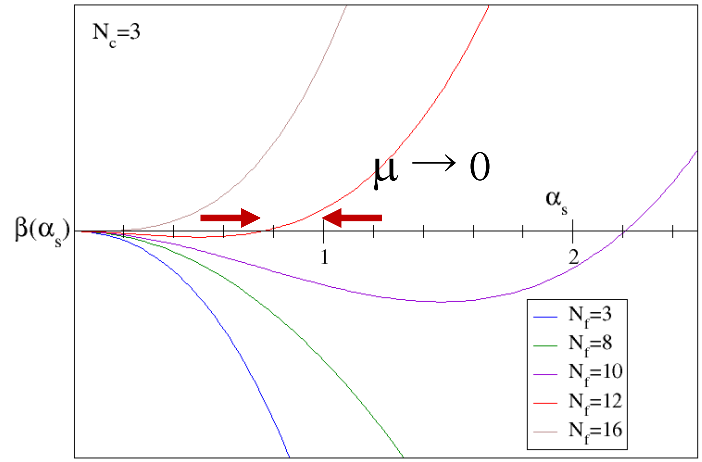
- **Caswell-Banks-Zaks IRFP**

In 2-loop perturbation, beta function

$$\beta(g^2) \equiv \frac{\partial \alpha}{\partial \ln \mu^2} = -\beta_0 \alpha_s^2 - \beta_1 \alpha_s^3, \quad \alpha_s = g^2/(4\pi)$$

$$\beta_0 = \frac{1}{4\pi}\left(\frac{11}{3}N_c - \frac{2}{3}N_f\right),$$

$$\beta_1 = \frac{1}{16\pi^2}\left(\frac{34}{3}N_c - \left\{\frac{13}{3}N_c - \frac{1}{N_c}\right\}N_f\right)$$



IRFP ($N_c=3$) in N_f^{cr} ($\simeq$ 8)< N_f <N_f^{af} ($\simeq$17)

Caswell (1974), Banks and Zaks (1982)

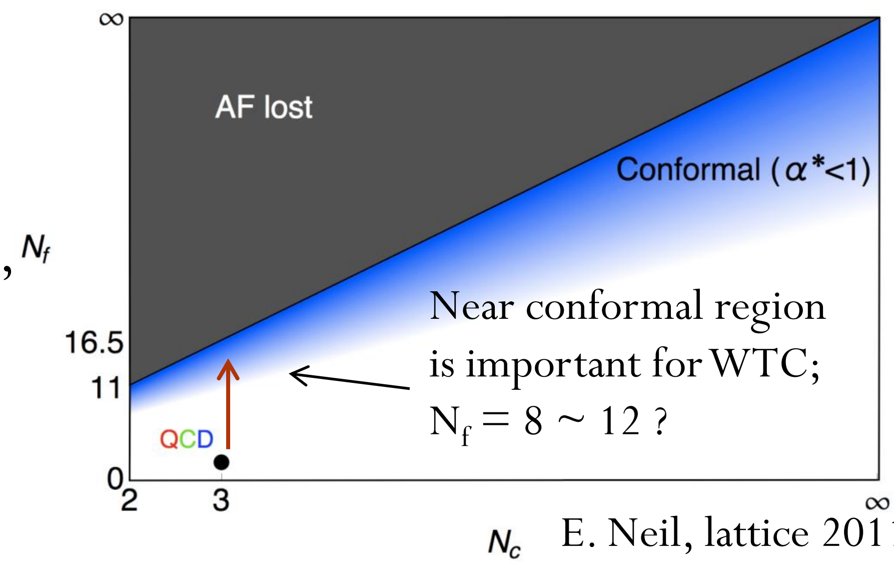
- m = 0 (fundamental rep.)
- $g(\mu \to 0) = g^*$, IR conformal

$\Rightarrow$ "conformal window"

Chiral symmetry will not be broken,

and 1st order transition occurs ?

This perception will be clear in the lattice calculation.



E. Neil, lattice 2011

Motivation

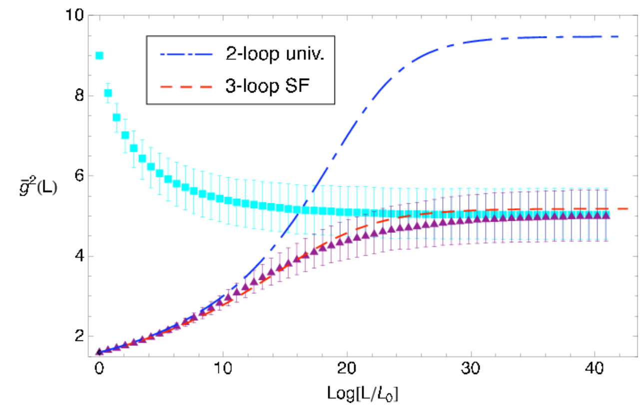
- **Lattice study in 12 flavor**
  - Appelquist, et al. (2008,2009)
    - Measurement of the running coupling in SF scheme in staggered fermion
    - See plateau region in $g^2 \simeq$ 4-5 that is IRFP

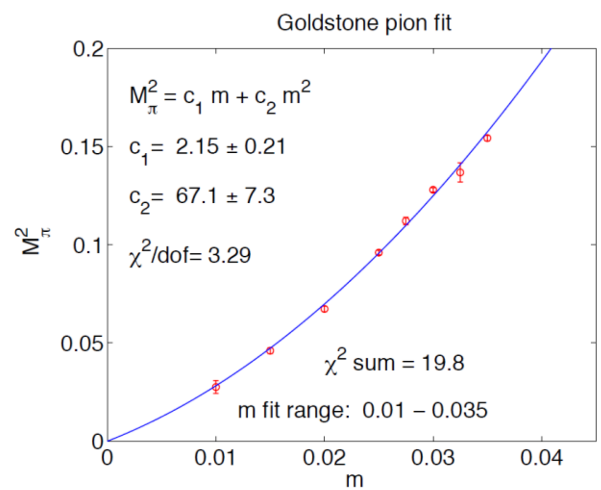
  $\Rightarrow$ $\underline{N_f = 12 \text{ is conformal}}$

  - Fodor et al. (2009,2011)
    - Goldstone pion and composite hadron spectrum in the improved staggered fermion
    - Goldstone pion behaves like χSB, and $\langle\bar{\psi}\psi\rangle \neq 0$

  $\Rightarrow$ $\underline{N_f = 12 \text{ is in broken phase}}$

  - A. Hasenfratz, (2010,2011)
    - Monte Calro renormalization group (MCRG)
    - Beta function will cross zero from negative to positive $\Rightarrow$ $\underline{N_f = 12 \text{ is conformal}}$





$\boxed{\text{Need confirmation in 12 flavor with different scheme}}$

New scheme

- **Twisted Polyakov loop scheme**
  - Different lattice scheme from SF and MCRG
  - Avoid a fake (unphysical) fixed point
  - Free from $O(a)$ discretization error
  - Practically cheaper cost than Wilson loop scheme Bilgici et al. (2009), Holland (2009)

    Not need large volume, zero fermion mass simulation
  - Consistency check with Wilson loop scheme in quenched QCD has been already done. Itou (2010)
  - Measure the renormalized running coupling in the continuum limit

    Obtain anomalous dimension of coupling constant (universal object)
  - First target on 12 flavor QCD in fundamental representation

Twisted Polyakov loop scheme

Polyakov loop in twisted boundary

- **Twisted boundary condition**
  - To avoid degeneracy of vacua ("toron") ‘tHooft (1979), Gonzalez-Arroyo (1988)
  - Suppression of Z(3) phase transition

$\Rightarrow$ well defined perturbative expansion of Wilson loop Divitiis, et al. (1994)

For gauge field (link variable):

$$U_\mu(x+\hat{\nu}L/a) = \Omega_\nu U_\mu(x)\Omega_\nu^\dagger, \quad \Omega_\nu\text{: twist matrix}$$

$$\Omega_1\Omega_2 = e^{i2\pi/3}\Omega_2\Omega_1,\ \Omega_\nu\Omega_\nu^\dagger = 1,\ \Omega_\nu^3 = 1,\ \mathrm{Tr}\,\Omega_\nu = 0$$

For fermion field:

$$\psi_{\underbrace{\alpha}_{\text{spinor}}}^{\overbrace{a}^{\text{smell}}\overbrace{b}^{\text{color}}}(x+\hat{\nu}a/L) = e^{i\pi/3}(\Omega_\nu)^{aa'}\psi_\alpha^{a'b'}(x)(\Omega_\nu^\dagger)^{b'b}$$

Parisi, (1983)
Wong, et al. (2006)

Add "smell" degree of freedom being same as color, corresponding to extra flavor, in order to avoid inconsistency with translational invariance.

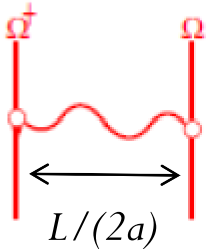
Using staggered fermion, flavor number is $N_f = 4 \times 3 = 12$

Polyakov loop in twisted boundary

- **Ratio of Polyakov loop correlator**

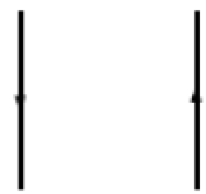
$$P_x(y,z,t) = \mathrm{Tr}\left[\prod_x U_x(x,y,z,t) \underbrace{\Omega_x}_{\text{Gauge inv.}} \underbrace{e^{i2\pi/3}}_{\text{Trans. inv.}}\right], \quad P_z(x,y,t) = \mathrm{Tr}\left[\prod_z U_z(x,y,z,t)\right]$$

$$\langle P_x(t = L/(2a)) P_x^*(0) \rangle$$

$=$ 

$\sim k g_0^2 + \mathcal{O}(g_0^4)$

$$\langle P_z(t = L/(2a)) P_z^*(0) \rangle$$

$=$ 

$\sim 1 + \mathcal{O}(g_0^4)$

Divitiis (1994)

$$(g_{\mathrm{TP}}^{\mathrm{lat}})^2 = \frac{1}{k} \frac{\langle \sum_{y,z} P_x(y,z,L/(2a)) P_x^*(0,0,0) \rangle}{\langle \sum_{x,y} P_z(x,y,L/(2a)) P_z^*(0,0,0) \rangle}, \quad (g_{\mathrm{TP}}^{\mathrm{lat}})^2\big|_{\mathrm{tree}} = g_0^2$$

$$k = 0.0318471147\cdots + 0.00453(a/L)^2$$

- $(g_{\mathrm{TP}}^{\mathrm{lat}})^2$ starts from log divergence (renormalized), <u>no linear divergence</u> due to cancelation between numerator and denominator.

  $\longrightarrow$ free from *O(a/L)* discretization error

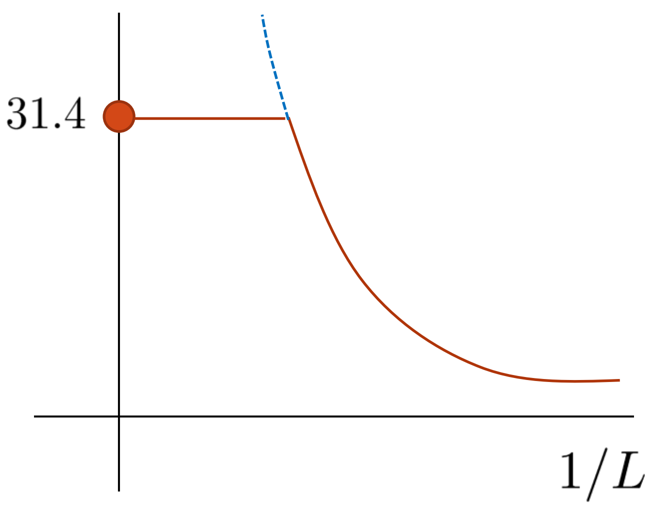
Polyakov loop in twisted boundary

- **Behavior in IR region**
  - In the infinite size, Polyakov loop correlator does not depend on boundary, thanks to cluster theorem, then

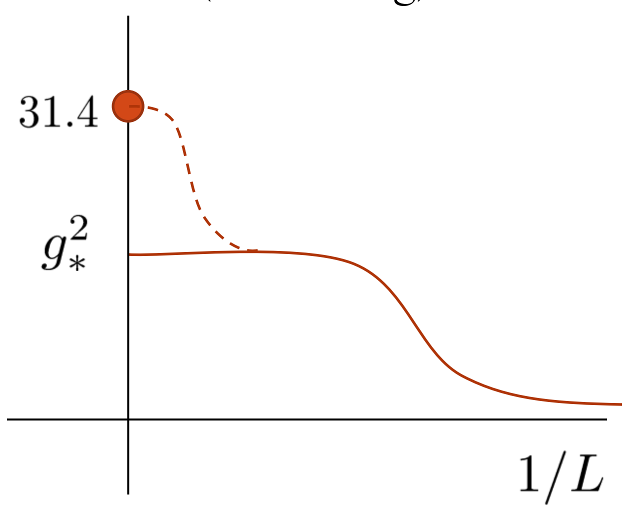
$$\lim_{L\to\infty} \frac{\langle \sum_{y,z} P_x(y,z,L/(2a))P_x^*(0,0,0)\rangle}{\langle \sum_{x,y} P_z(x,y,L/(2a))P_z^*(0,0,0)\rangle} = 1 \quad \Rightarrow \quad [g^2_{\mathrm{TPL}}]_{\mathrm{max}} \sim \frac{1}{k} \simeq 31.4$$

  - The above value is maximum in the IR limit. It turns out that it is able to distinguish IRFP from other (e.g. QCD-like) theory.

QCD-like (no-IRFP or $g_*^2 \geq 31.4$)



IRFP (or walking)



Running coupling constant

- **Step scaling method**
  - Inverse of volume is interpreted as a scale.
  - Changing the volume gives the stepping behavior of running coupling
  - Start from reference point: u_0

    which is weak coupling constant in the continuum theory
  - Set a step parameter: s

    Large step parameter is recommended to investigate deep IR region and suppress correlation
  - Step scaling process:

    1. $u_0 = g^2_{\rm TPL}(1/L)$,
    2. $(g^{\rm lat}_{\rm TPL})^2(a, a/L) = u_0, \quad \Sigma(a, a/(sL))) = (g^{\rm lat}_{\rm TPL})^2(a, a/(sL))$
    3. $\sigma(u_0) = \lim_{a \to 0} \Sigma(a/(sL), a)$,
    4. $u_1 = \sigma(u_0)$ return to 1.

    e.g. s = 2, aL = 4,6,8 $\rightarrow$ saL = 8,12,16, and taking continuum limit with several beta simulation, we can get step scaling in the continuum theory.
  - If an IRFP exists in u=u* , see $\sigma(u^*)/u^* = 1$

Nf=12 results

Nf=12, staggered fermion

- Dynamical fermion with "smell" in HMC

  4 flavor(default) × 3 smell = 12 flavor effectively

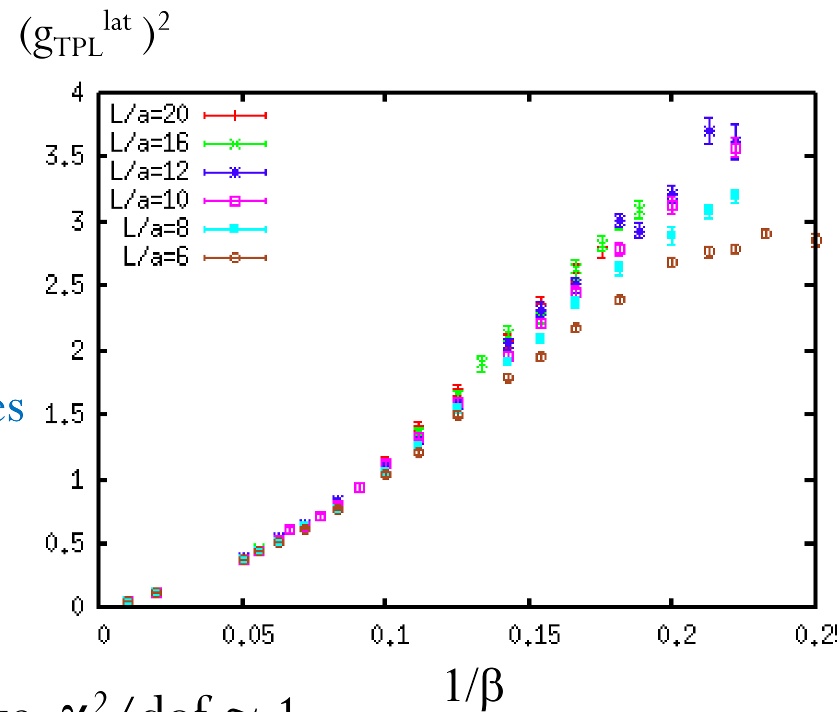
- Simulation on the massless point, thanks to twisted boundary

  No need of chiral extrapolation

- Parameters:
  - 4 lattice size, $L/a = 6,8,10,12$

    and $sL/a = 9,12,15,18$ with $s = 1.5$

    Interpolating points
  - $1/\beta = 0.01$ -- 0.25
  - 8k(high β) – 897k(low β) trajectories



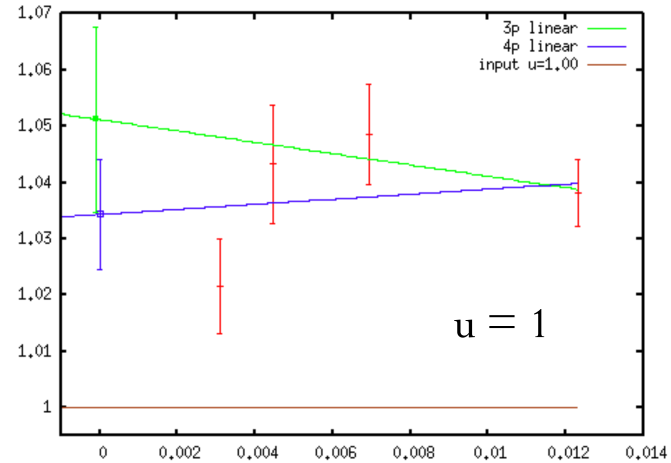
- Fit function for interpolation

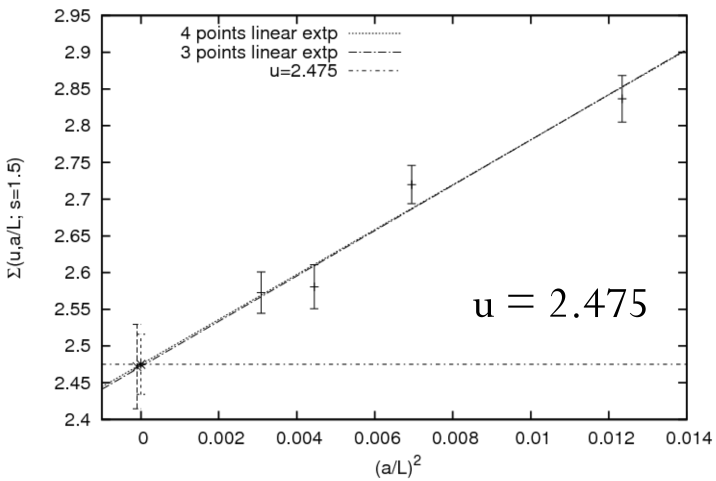
$$(g_{\mathrm{TPL}}^{\mathrm{lat}})^2 = \frac{6}{\beta} + \sum_{i=1}^{N} C_i(a/L)\beta^{-i-1}$$

with $N = 3 - 5$ depending on lattice size, $\chi^2/\mathrm{dof} \sim 1$

Nf=12, staggered fermion

- **Continuum limit**
  - After s step, we need to take the continuum limit. In TPL scheme, $O(a/L)$ error is absent.
  - We perform a linear extrapolation for $(a/L)^2$ using data in $sL/a = 9,12,15,18$.
  - Linear function well describes lattice data.
  - 3 and 4 points fit are not so different. It turns out that systematic error of scaling violation is not so large.





Nf=12, staggered fermion

- **Running coupling constant in TPL scheme**
  - Behavior of ratio, σ(u)/u vs u
    - From u ≃ 0.5, σ(u)/u grows up rather than 2-loop, and around u ≃ 1.3 it declines to 1.
    - u ≃ 2.5 this ratio crosses 1, and it will be conformal point.
    - Anomalous dimension of coupling γ, which is slope at u*

$$\gamma = 0.52^{+27}_{-24}$$

2-loop: γ ~ 0.36

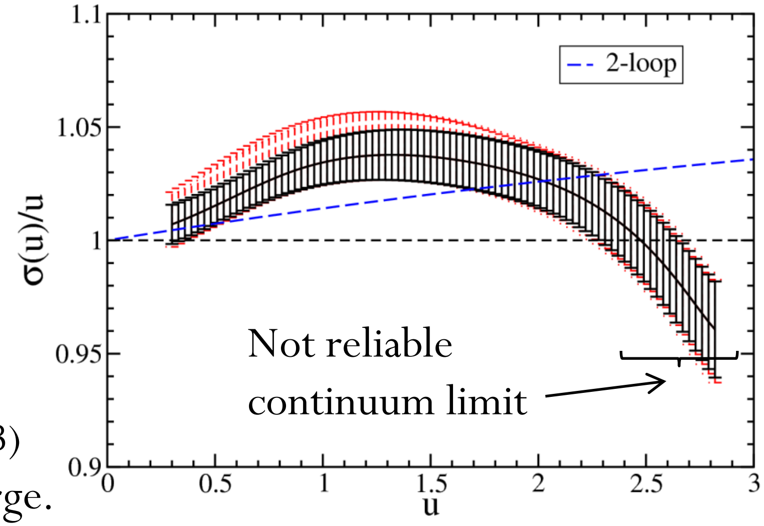
SF scheme: γ = 0.13(3)

consistent with 2-loop result, but error is still large.

⇒ future works

Black solid : statistical error

Red dashed: total error



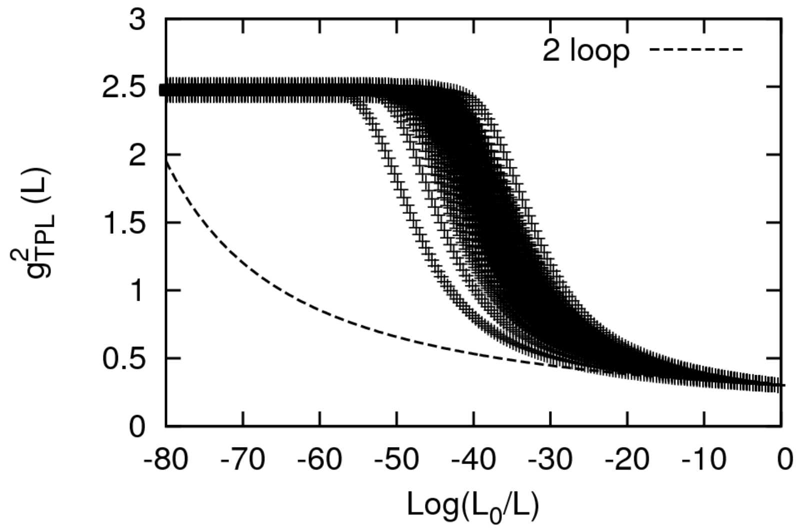
  - Running behavior

UV region: Consistent with 2-loop

IR region:

Observe a unique plateau region for every distribution of JK ensembles, and then

$$(g^2_{\rm TPL})^* = 2.48(18)(^{+7}_{-8}) \ll [g^2_{\rm TPL}]_{\rm max}$$



Summary and outlook

- Simulation of $N_f = 12$ SU(3) gauge with fundamental fermion using "Twisted Polyakov loop" scheme.
- High statistics and rigorous lattice calculation at massless point
  - Take the conitnuum limit, and control scaling violation
- Running coupling constant slows down and stops in $(g^2_{\mathrm{TPL}})^* = 2.48(18)(^{+7}_{-8})$
- Our results establish that

  Nf = 12 QCD (fund.) has an infrared conformality.

- On-going works:
  - Mass anomalous dimension in this scheme

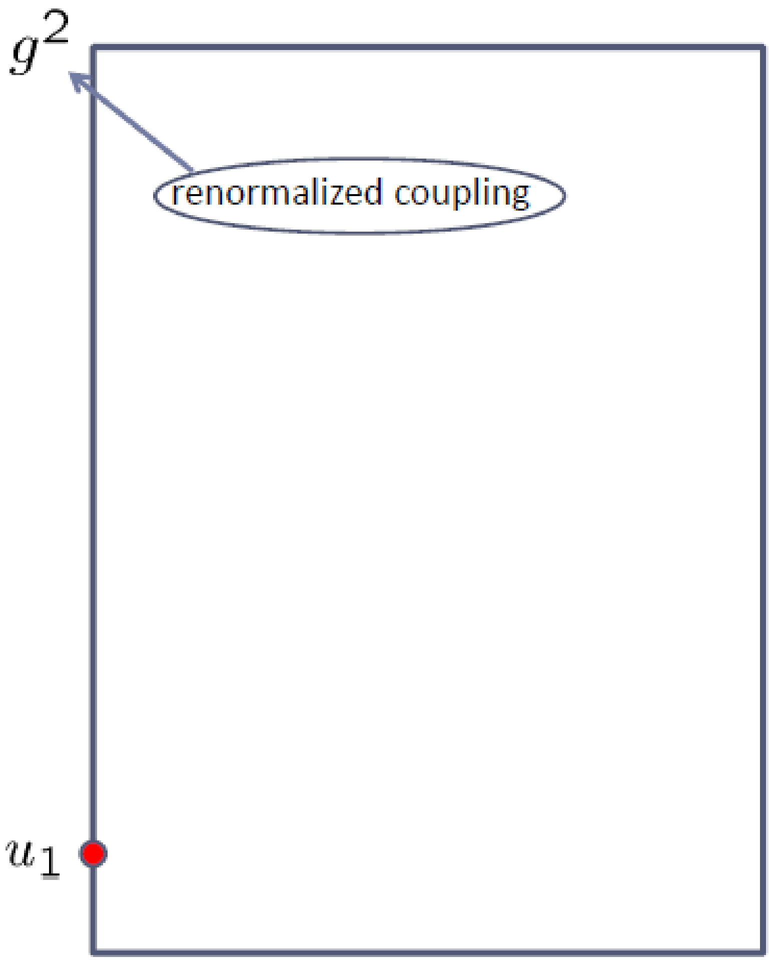
    Obtained from correlator of Goldstone pion
  - SU(2) fundamental in $N_f = 8$, this is almost done

- Appelquist, et al.
- Haesenfratz
- Aoki (KMI)
- Pallente, et al.

Backup

Example

Step scaling



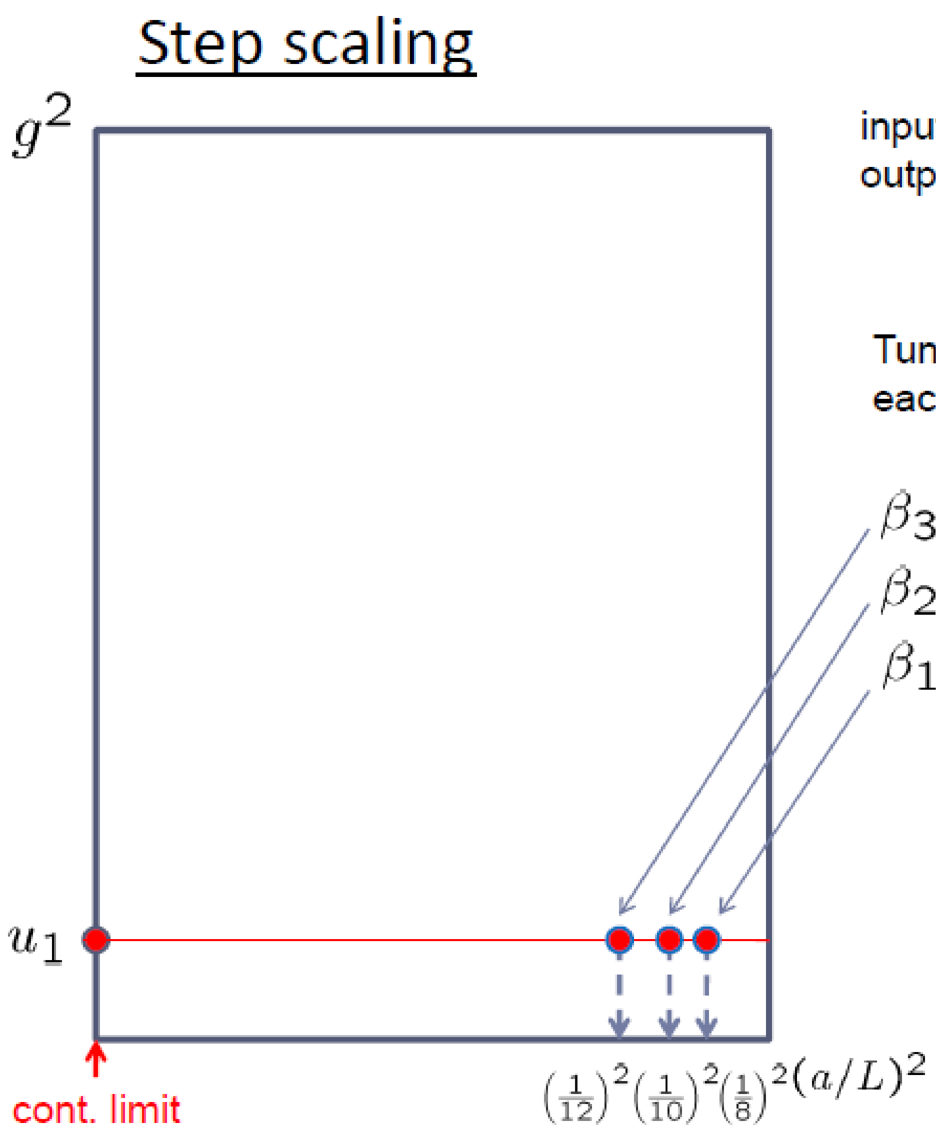
input $s = 1 : L = 8, 10, 12$

output $s = 2 : L = 16, 20, 24$

Choose a value of the renormalized coupling constant at energy scale ($\mu = 1/L_0$)

$(a/L)^2 \leftarrow$ inverse of lattice size

Example

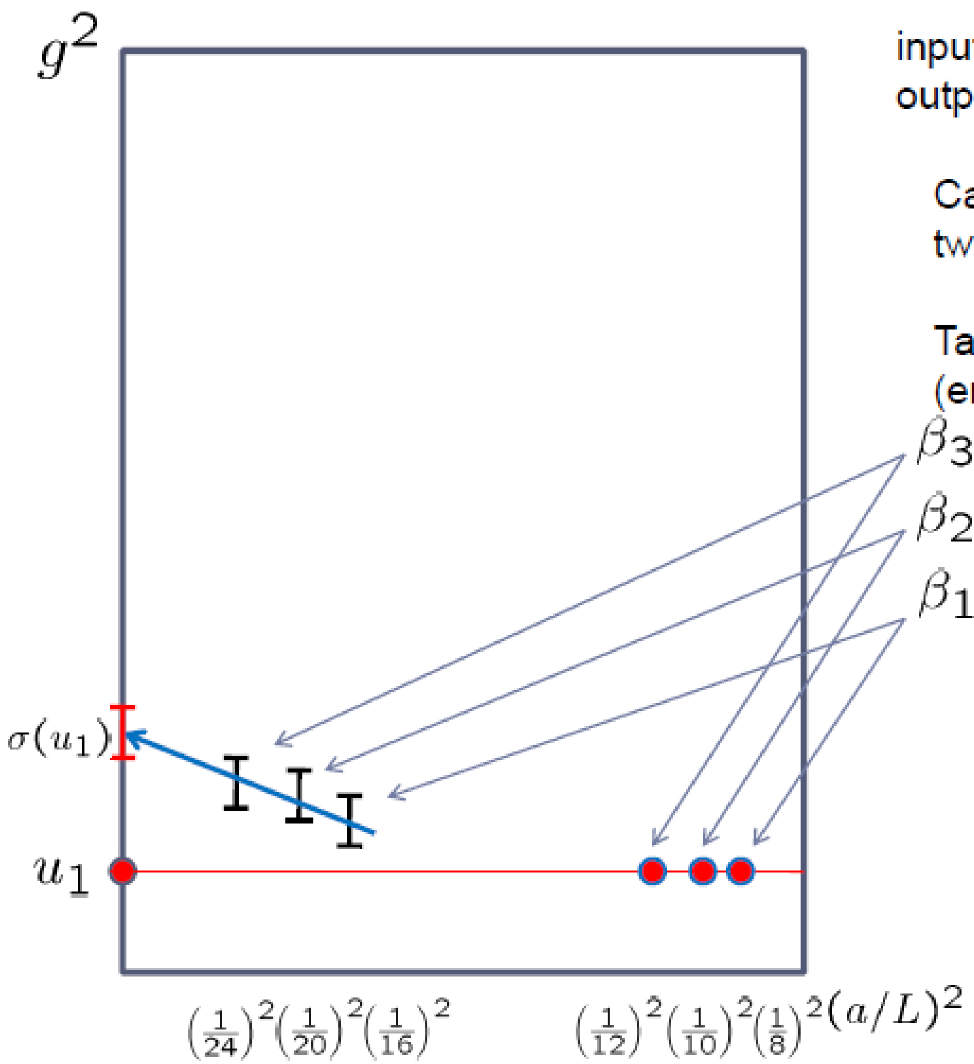


input $s = 1 : L = 8, 10, 12$
output $s = 2 : L = 16, 20, 24$

Tune the beta (bare coupling) for each small lattice size.

Example

<u>Step scaling</u>



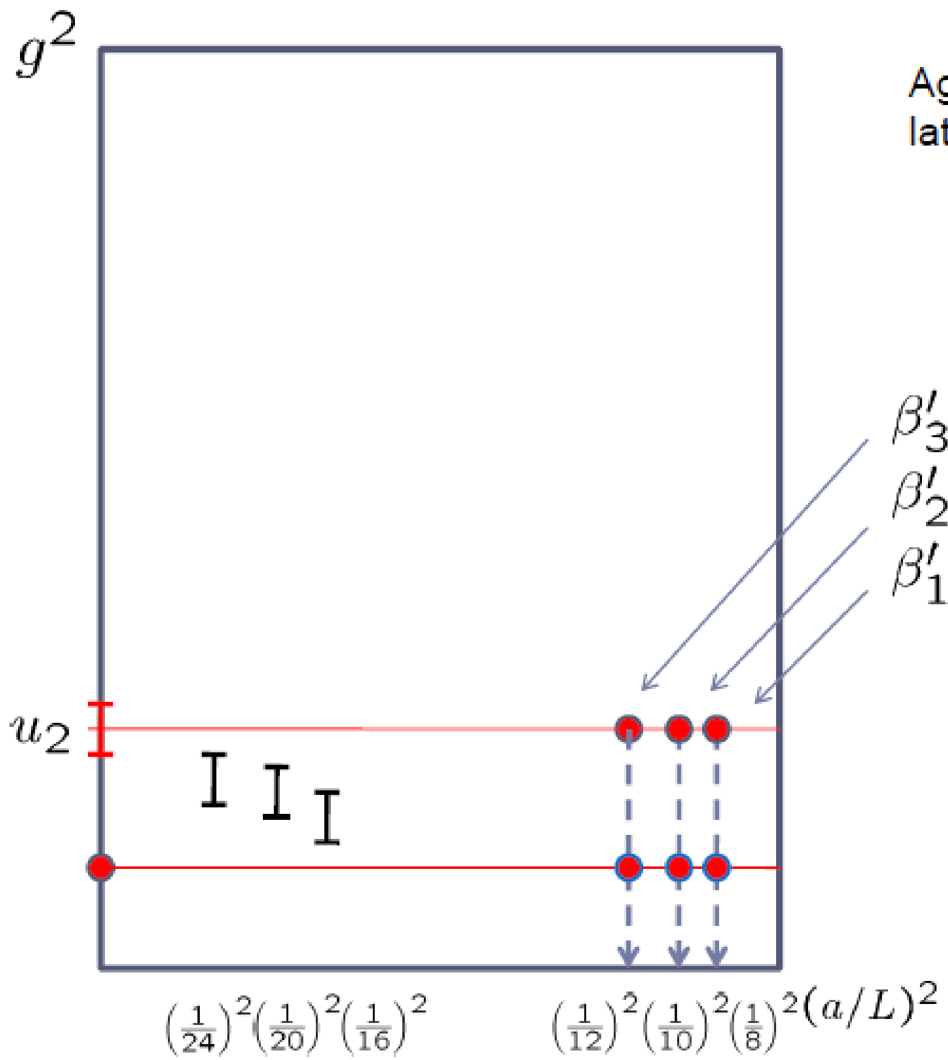
input $s = 1 : L = 8, 10, 12$
output $s = 2 : L = 16, 20, 24$

Carry out the simulation for the twice size of lattice.

Take the continuum limit (energy scale $\mu = 1/2L_0$)

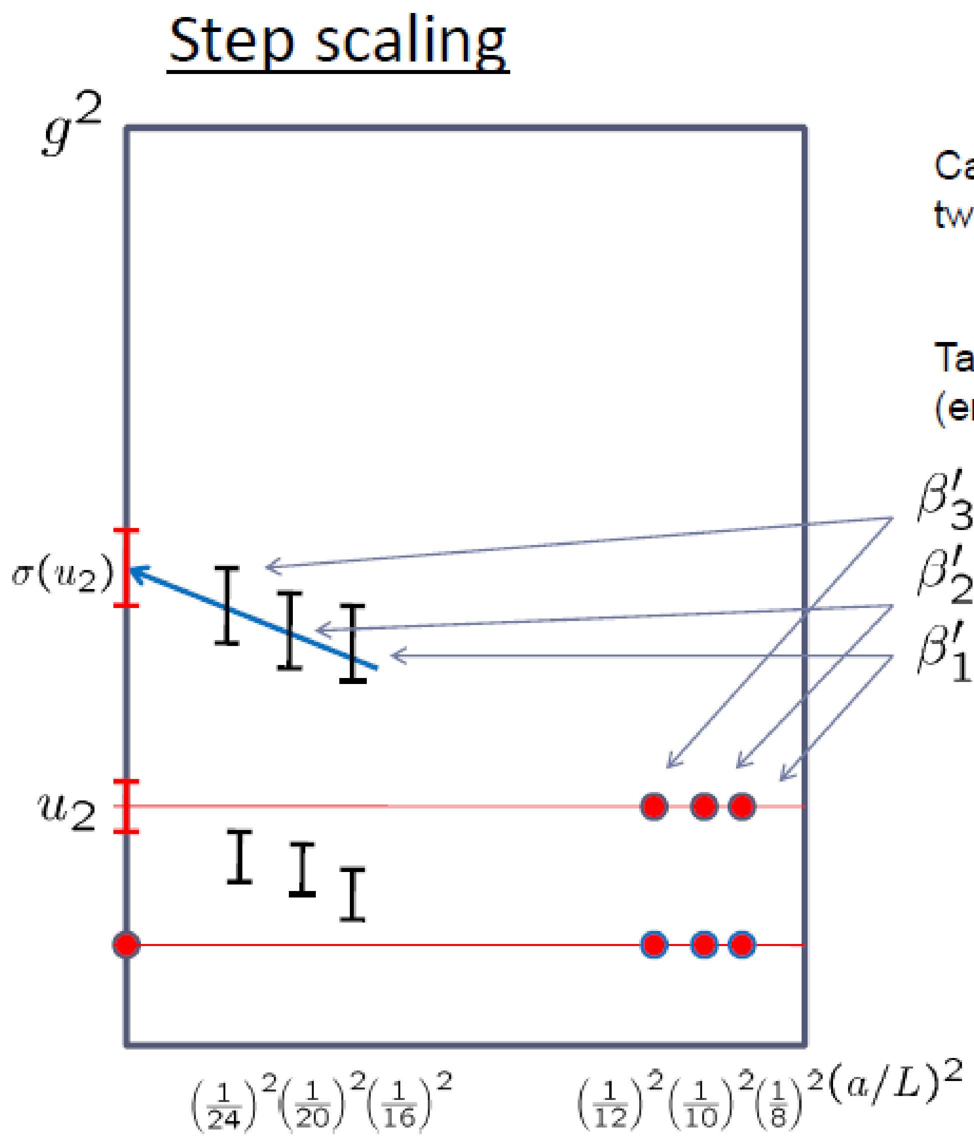
Example

Step scaling



Again, tune the beta at small lattice size

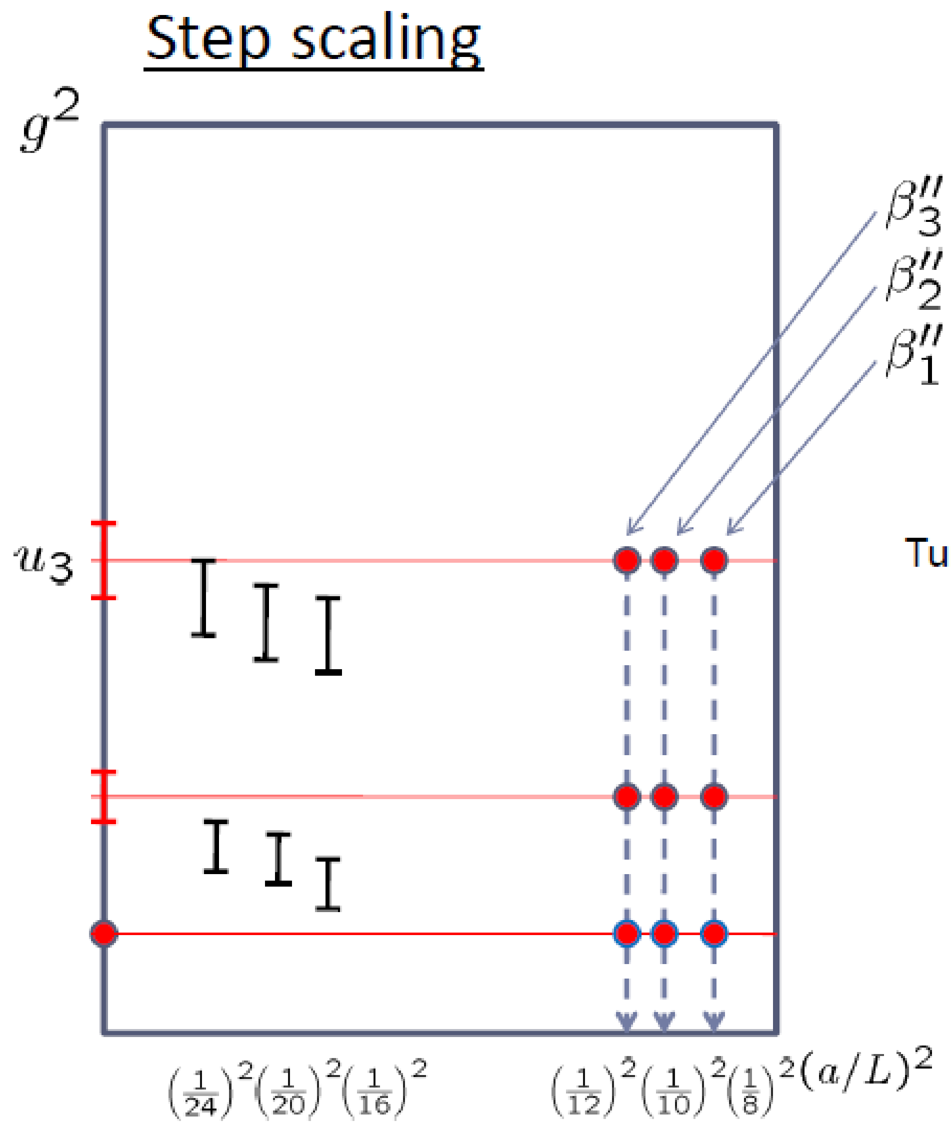
Example



Carry out the simulation for the twice size of lattice.

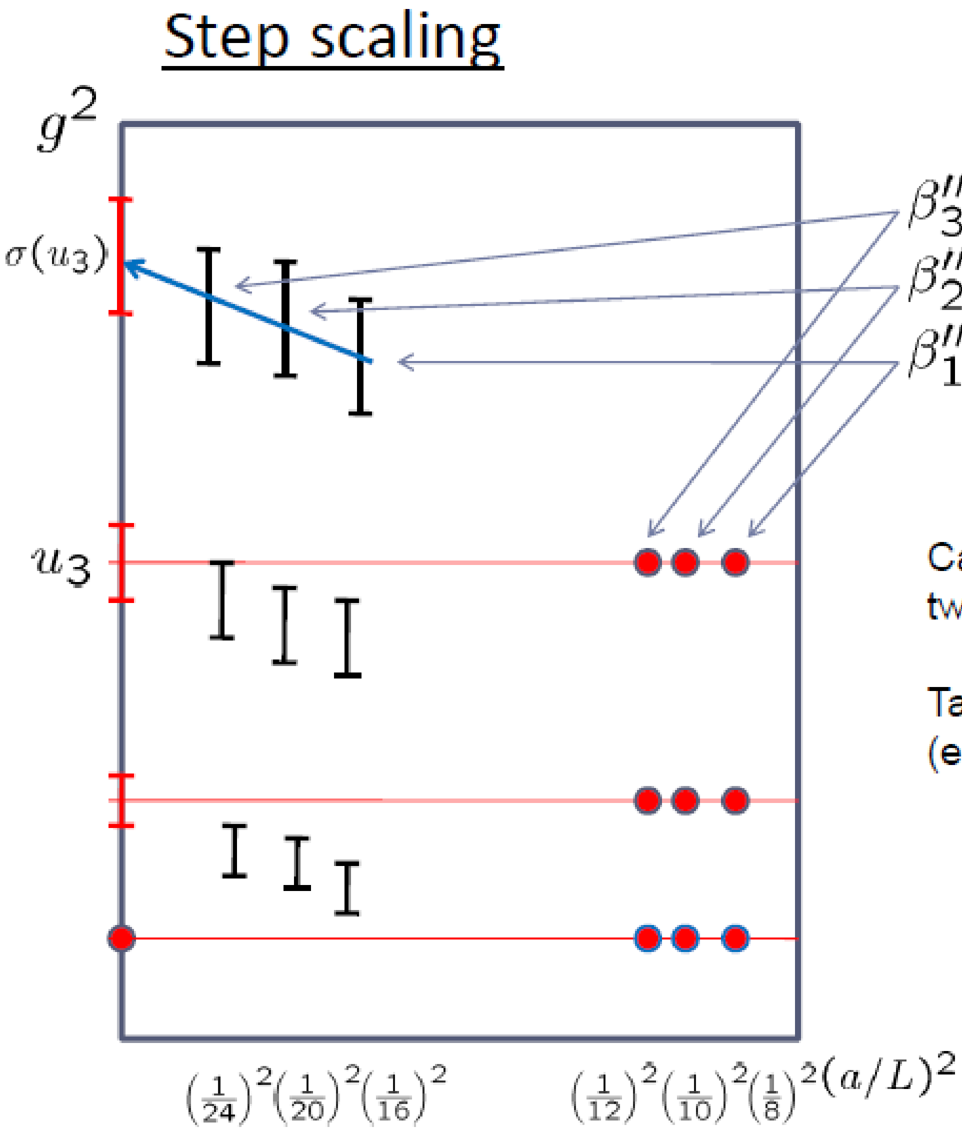
Take the continuum limit
(energy scale $\mu = 1/4L_0$)

Example



Tune the beta at small lattice size

Example

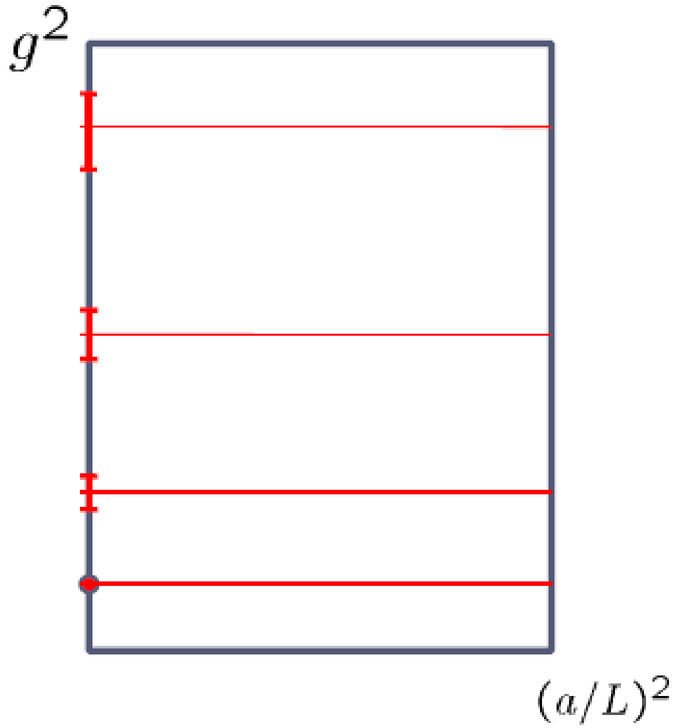


Carry out the simulation for the twice size of lattice

Take the continuum limit (energy scale $\mu = 1/8L_0$)

Example

Step scaling



We obtain the scaling of the running coupling.